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AFOSR-TR. 86-0552

Stability Analysis of Interconnected Random-Access Networks

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Abstract

We consider the interconnection of two multiple-access/broadcast networks, each of which connects a large population of bursty users via a packet-switched, random-access channel. In each network a station, called bridge node, receives internetwork packets from the local users and forwards them to the bridge node of the destination network via a point-to-point link; the bridge node of the destination network places these internetwork packets in its queue for subsequent broadcasting to the local users. We consider two ways of multiplexing the local traffic and the internetwork traffic: a) contention multiplexing, b) channel division multiplexing. Under contention multiplexing, the bridge node uses the same random-access channel that the local users use, and therefore it participates in the contention. Under channel division multiplexing, the channel in each of the two networks is subdivided into a node subchannel, used exclusively by the bridge node, and a random-access channel, used by the local users. Assuming that the input traffic in network i, i=1,2, is Poisson with intensity λ_i, the stability region of the interconnected system is defined as $S = \{(\lambda_1,\lambda_2):$ the packet delay is finite with probability one$\}$. We develop an analysis method for determining a subset of S, and we give explicit results when the Stack random-access algorithm is used to resolve conflicts at the local level.

86 9 15 108

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
UNCLASSIFIED	

2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT
NA	Approved for Public Release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
NA	

4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
UCT/DEECS/TR-86-6	AFOSR·TR· 86-0552

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
University of Connecticut		AFOSR/NM

6c. ADDRESS (City, State and ZIP Code)	7b. ADDRESS (City, State and ZIP Code)
U-157, 260 Glenbrook Rd. Storrs, CT 06268	Bldg. 410 Bolling AFB, DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NM	AFOSR-83-0229

8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS.			
Bldg. 410 Bolling AFB,DC	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
	6.1102F	2304	A5	

11. TITLE *(Include Security Classification)* Stability Analysis of Interconnected Random Access Networks

12. PERSONAL AUTHOR(S) L. Merakos, L. Georgiadis, and C. Bisdikian

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Interim	FROM 7/1/85 TO 6/30/86	April, 1986	

16. SUPPLEMENTARY NOTATION

17.	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Random Access, Network Interconnection, Queueing
XXXX	XXXXXXXXXX	XXXXXX	

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED/UNLIMITED ☑ SAME AS RPT. ☐ DTIC USERS ☐	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Prof. P. Papantoni-Kazakos	(203) 486-2867	AFOSR/NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

I. INTRODUCTION

Network interconnection is a necessity when we want to provide full connectivity to geographically separated networks, but it can also be used to enhance performance and reliability in a fully connected network by appropriately clustering its users into self-contained interconnected subnetworks.

In this paper, we consider the interconnection of two multiple-access/broadcast networks, each of which connects a large population of bursty users via a packet-switched, random-access, broadcast channel. Both networks are assumed single-hop; that is, a transmission from any one user can be heard by all other users in its own network. However, no user in one network can hear the transmission of a user in the other network. Internetwork communication is accomplished by means of a point-to-point link, called bridge link, connecting two stations, one in each network, called bridge nodes. The function of a bridge node is twofold: a) It relays internetwork traffic, (i.e., traffic generated in one network and destined to the other), to the bridge node of the other network via the bridge link. b) It broadcasts internetwork traffic received from the other bridge node to the users of its own network, (local users). The setting is illustrated graphically in Figure 1, and can be used to model several networking scenaria. For example, the bridge link might be a satellite or a microwave link connecting widely separated local terrestrial packet-radio networks, or a gateway connecting collocated local cable networks.

We will adopt the infinite population, Poisson user model for each of the two networks. Users transmit messages in the form of fixed length packets. Packets are generated by all users of network i, i=1,2, according to an independent Poisson process at a rate of λ_i packets per unit time. Since the channel in each network is shared, simultaneous transmissions result in packet collisions. Collided packets are assumed destroyed and they have to be retransmitted at some later time. Packet transmissions and retransmissions are coordinated by a random-access algorithm (RAA) (In this paper we will adopt the Stack algorithm, [1, 2, 3, 4]).

A-1

A packet generated in network i, i=1,2, may be an <u>intranetwork packet</u> or an <u>internetwork packet</u>. An intranetwork packet has a final destination in the network in which it was generated. Therefore, an intranetwork packet leaves the system after its first successful transmission (one hop). An internetwork packet generated in network 1(2) was a final destination in network 2(1). Therefore, such a packet leaves the system only after it has been successfully transmitted in both networks. This is accomplished in three hops, as follows. Upon successful transmission in the network that generated it, (first hop), an internetwork packet is received by the local bridge node, which, then, transmits it to the bridge node of the destination network via the bridge link, (second hop). The bridge node of the destination network stores the packet for subsequent broadcasting to local users; upon successful broadcasting, (third hop), the packet leaves the system.

A bridge node must be able to distinguish between intranetwork and internetwork packets, in order to know which packets to transmit over the bridge link. We will assume that this information can be extracted from the address field in each packet. Furthermore, since packets may arrive faster than they can be retransmitted, the bridge node must contain enough buffer space in which packets are temporarily queued. The queueing model of a bridge node is illustrated in Figure 2. It consists of two queues, referred to as the <u>network queue</u> and the <u>link queue</u>. The network queue contains the packets that were received on the bridge link and are waiting to be broadcast to the local users; the link queue contains the packets that were received on the local random-access channel and are waiting to be transmitted on the bridge link.

The volume of traffic that can be supported before the interconnected system becomes congested, and the delay that a packet experiences until it reaches its final destination depend on how the available system resources (the two broadcast channels and the bridge link channel), are allocated among the network users and the bridge nodes.

For the bridge link channel we will assume that frequency or time division multiplexing provides two way communication between the bridge nodes. Also, we will assume that transmissions on the bridge link do not interfere with transmissions on the broadcast channels.

In each of the two broadcast channels we have user transmissions originating from the local users and coordinated by the underlying RAA, and node transmissions that are broadcast from the bridge node to the local users. For the multiplexing of the user and node traffic on the broadcast channel we will consider the following possibilities: a) Contention Multiplexing, b) Channel Division Multiplexing. Under contention multiplexing, the bridge node uses the same random-access channel that the local users use, and, therefore, it participates in the contention. Under channel division multiplexing, the channel is divided into a node subchannel, which is used exclusively for node transmissions, and a random-access user subchannel, which is used for user transmissions; the channel division may be done either in the frequency domain or in the time domain.

The flow of packets in the interconnected system is shown in a block diagram form in Figure 3. The queues designated as DQ^1 and DQ^2 are distributed, and can be thought of as abstract storage devices containing the packets that have been generated by various network users but have not been successfully transmitted yet. Queue Q^i is the network queue of bridge node $i, i=1,2$. For simplicity in the analysis, the link queues of the bridge nodes have been eliminated from the block diagram of Figure 3. That is, we assume that packets experience no queueing delay at the transmitting end of the bridge link. This assumption implies that for the link queue the service time is less than the minimum interarrival time, which is true if the packet transmission time in the bridge link is less than the packet transmission time in the local random-access channel. The above assumption is not critical, since the system model with the link queues included can be analyzed using the analysis techniques to be developed for the model in Figure 3.

Under contention multiplexing the interconnected system is a system of four interacting queues with state-dependent service time distribution. Queuing problems of this kind have been known to be hard to treat analytically. What makes things worse in our model is that queues DQ^1, DQ^2 are distributed, and that queues Q^1, Q^2 have non-independent interarrival time processes with state-dependent distributions.

Recognizing the analytical difficulties, we will only be concerned with the determination of the region in the (λ_1, λ_2) plane in which the system is stable. The system is called stable if the packet delay is finite with probability one. (A more precise stability definition for the system and its constituent queues will be given in Section II.B). The method of analysis uses an auxiliary system of queues that dominates the given one, (in the sense defined in Section II.D).

The stability analysis of the system under channel division multiplexing is less difficult, since, by channel division, the four-queue system is decomposed into the two tandem queue systems, (DQ^1, Q^2) and (DQ^2, Q^1), which evolve independently of each other.

The organization of the paper is as follows. Section II presents the stability analysis of the system under contention multiplexing, for specific transmission policies for the bridge nodes and the network users. The system with channel division multiplexing is studied in Section III. Section IV provides some concluding remarks. The paper ends with an Appendix which includes results used in the main body of the paper; some of these results are of independent interest, since they are applicable to more general contexts than that of multiple access communication networks.

II. CONTENTION MULTIPLEXING

A. Channel Model and Transmission Policies.

Under contention multiplexing, the bridge node and the users in each of the two networks share a common random-access channel. A simple channel model is considered. The channel time is divided into slots of length equal to the packet transmission time, which is taken as the unit of time. Slot n denotes the interval $[n, n+1)$, $n = 0, 1, 2 \ldots$

The bridge node and the users may initiate a packet transmission only at the beginning of a slot. If more than one packet are transmitted within the same slot, then a packet collision occurs. It is assumed that a collision results in complete loss of the information included in all the involved packets; thus, retransmission is then necessary. If only one packet is transmitted in a slot, it is received with no errors, and the transmission is said to be successful.

Regarding the interconnected system, we will make the following assumptions: The system starts operating at time n=0 empty of packets. The packet transmission time (slot size) is the same in both broadcast channels and the channels are slot synchronized; that is, the interval $[n,n+1)$, $n=0,1,2,\ldots$, corresponds to slot n in both channels. The propagation delays in the bridge link and the broadcast channels, and any processing delays at the bridge nodes and the users are zero. (The above assumptions are not necessary, but allow us to avoid undue complication in the notation and the statements of the results.)

Throughout the paper we will use the letter i as a superscript or subscript to signify quantities that refer to network i, i=1,2. We will also use i', where i'=1 if i=2, and i'=2 if i=1.

Let the packets generated in network i be indexed according to their time of generation and define the random variable

$$\rho_\xi^i = \begin{cases} 1 & \text{if the } \xi\text{-th packet generated in network i is an internetwork packet} \\ 0 & \text{otherwise} \end{cases}$$

It will be assumed that the sequence $\{\rho_\xi^i\}$ is i.i.d., with $\Pr(\rho_\xi^i=1)=p_i$, and independent of any other process in the system.

Next we specify the transmission policies for the bridge nodes and the users.

Transmission Policy for the Bridge Nodes

For bridge node i, define the i.i.d. discrete-time process $\{T_n; n \geq 1\}$ with

$$T_n^i = \begin{cases} 1 & \text{with probability } \pi_i \\ 0 & \text{with probability } 1-\pi_i \end{cases}$$

, and assume that $\{T_n^i; n \geq 1\}$ is independent of any other process in the system.

At the beginning of each slot n, at which it transmitted a packet, the bridge node i transmits a packet from its queue if and only if $T_n^i = 1$.

We assume that by the end of a slot, in which it transmitted a packet, the bridge node can determine whether the packet was successfully transmitted or collided with local packets. A packet departs the queue if and only if it is successfully transmitted. The probabilities π_1, π_2 are design parameters.

The packet priorities, i.e., the service discipline at the queue of a bridge node, can be arbitrary but specified, although not necessarily the same for both bridge nodes.

Transmission Policy for the Users

In each of the two networks users transmit their packets according to a RAA. Among the plethora of RAAs that have been proposed for a single-hop environment, we consider the n-ary Stack Algorithms (SA_n), [1,2,3,4]. We opted for this particular algorithm because of its simplicity and relatively high performance. The method of analysis, however, can be applied to other popular RAA's, (e.g., Aloha-type algorithms, Tree Search-type algorithms).

The SA_n is a "limited feedback sensing" algorithm that uses binary feedback of the "Collision-Non-Collision" (C-NC) type. A user monitors the channel activity for acquisition of feedback information only while it has a packet to transmit (limited feedback sensing); we assume that, at the end of each slot, a user that monitors the channel can determine whether that slot contained a collision or not (C-CN). Note that in the interconnected system, a collision may have been caused by a node transmission.

Let F_n^i denote the binary feedback corresponding to slot n, of random-access channel i. Accordingly, $F_n^i = NC$, and $F_n^i = C$ represent a non-collision and a collision

in slot n, respectively. The "limited feedback sensing" assumption implies that for the transmission of a packet arrived during slot (n-1) no knowledge of the channel feedback history, $\{F_k^i; k<n\}$, is necessary.

For the transmission of its packet ξ a user utilizes a counter, whose indication at the beginning of slot n is denoted by $I_n^i(\xi)$, and it applies the following set of rules that define the SA_n:

1. For a packet ξ arrived during slot (n-1) the user sets $I_n^i(\xi)=0$

2. Packet ξ is transmitted in slot n if and only if $I_n^i(\xi)=0$

   Packet ξ is successfully transmitted in slot n if and only if $I_n^i(\xi)=0$ and $F_n^i=NC$.

3. The updating of the counter indication $I_n^i(\xi)$ is done as follows:

   a) If $F_n^i=NC$ and $I_n^i(\xi)>0$, then $I_{n+1}^i(\xi)=I_n^i(\xi)-1$

   b) If $F_n^i=C$ and $I_n^i(\xi)=0$, then $I_{n+1}^i(\xi)=U_{n+1}^i(\xi)$, where $U_{n+1}^i(\xi)$ is a random variable uniformly distributed on $\{0,1,\ldots,n-1\}$, independent of any other variable in the system, and n is a design parameter, $n\geq2$.

   c) If $F_n^i=C$ and $I_n^i(\xi)>0$, then $I_{n+1}^i(\xi)=I_n^i(\xi)+n-1$

Under the SA_n the distributed queues DQ^i, i=1,2, in Figure 3 take the form of the "stack" shown in Figure 4. The stack is an abstract storage device consisting of an infinite number of cells, labelled 0,1,2.... At the beginning of each slot n the j-th cell of stack i contains all packets ξ such that $I_n^i(\xi)=j$; j=0,1,2,.... Packets are eventually successfully transmitted after moving through the cells of the stack in accordance with the rules defined above. To resolve conflicts, the algorithm splits uniformly the group of collided user packets in the first n cells of the stack. The integer n is an algorithmic parameter, whose value may be chosen for performance optimization, [2].

B. Some Related Random Variables and a Markov Chain

Let us define the following random variables:

B_n^i = the number of packets in stack i,(i.e., in DQ^i), at time n.

Q_n^i = the number of packets in Q^i at time n.

M_n^i = the number of packets generated by the users of network i during slot n.

M_n^i is Poisson distributed with intensity λ_i.

$$A_n^i = \begin{cases} 1 & \text{if a packet enters queue i, i=1,2, during slot n} \\ 0 & \text{otherwise} \end{cases}$$

Note that A_n^i =1 if and only if the n-th slot of channel i is busy with a successful user transmission of an internetwork packet. Figure 5 is an illustration to aid in the visualization of the random variables defined above; (node i, i=1,2, represents the i-th random-access channel; the links that are directed towards a node represent transmission attempts, while the outgoing links represent successful transmissions.) The variable B_n^i can be expressed as follows:

$$B_n^i = \sum_{j=0}^{K_n^i-1} C_n^i(j)$$

where $C_n^i(j)$ denotes the number of packets in cell j of stack i at the beginning of slot n; (K_n^i-1) denotes the highest indexed cell of stack i that is possibly non-empty at the beginning of slot n. The integer K_n^i can be thought of as the position of a conceptual _pointer_ that moves through the cells of stack i according to the following rules:

$$K_1^i = 1$$

$$K_{n+1}^i = \max(K_n^i-1,1) \qquad \text{if } F_n^i = NC$$

$$K_{n+1}^i = K_n^i + n - 1 \qquad \text{if } F_n^i = C$$

Next define the random vector $Z_n, n=0,1,2,\ldots$, as follows:

$$Z_n = [Q_n^1, Q_n^2, (C_n^1(0),\ldots, C_n^1(K_n^1-1)), (C_n^2(0),\ldots,C_n^2(K_n^2-1)),K_n^1, K_n^2] \quad ; n>1 \quad (1)$$

$$Z_0 = [0, 0, (0), (0), 1, 1]$$

From the description of the system, it can be seen that the process $\{Z_n, n\geq 0\}$ is a Markov chain with countable state space $\Psi = N_0 \times N_0 \times \Theta \times \Theta \times N_0^+ \times N_0^+$, where $N_0 (N_0^+)$ is the set of non-negative (positive) integers, and Θ is the set of <u>finite</u> sequences of non-negative integers; i.e.,

$$\Theta = \{(k_1,k_2,\ldots,k_M) : M\epsilon N_0^+; k_1,k_2,\ldots,k_M\epsilon N_0\}$$

The ergodicity of $\{Z_n, n\geq 0\}$ will be of concern, but first we will give a precise definition of system stability.

C. <u>Stability Definition and Stability Region</u>

Let all packets generated in the system from the beginning of its operation be indexed according to their time of generation. The delay, δ_ℓ of the ℓ-th packet is defined as the time from the moment of its generation until the moment of its successful reception at its final destination. Our main interest will be in the asymptotic behavior of the distribution function of δ_ℓ, as well as that of the queue sizes B_n^i and Q_n^i, i=1,2. The asymptotic behavior will be described using the following stability definitions, [5].

A sequence of proper [1] random variables X_n with distribution functions F_n is called:

(a) <u>Stable</u>, if X_n converges in distribution to a proper random variable.

(b) <u>Substable</u>, if there are proper distributions G, H, such that

$G(x) \leq F_n(x) \leq H(x)$, for every n and x.

---

1. A random variable is proper if it has a proper distribution function.

A distribution function F is proper if $\lim F(x) = 1$, as $x \rightarrow +\infty$.

(c) Unstable, if it is not substable.

It can be shown that the definition of substability is equivalent to the following:

(b.1) The sequence F_n is relatively compact - i.e., each subsequence of F_n contains a stable sub-subsequence.

(b.2) $\lim_{x \to +\infty} F_n(x) = 1$, $\lim_{x \to -\infty} F_n(x) = 0$, uniformly in n.

(b.3) $\lim_{x \to +\infty} \lim_{n \to +\infty} \inf F_n(x) = 1$, $\lim_{x \to -\infty} \lim_{n \to +\infty} \sup F_n(x) = 0$

In the context of random-access networks, condition (b.3) has been used as the definition of stability in [8]; in the case of non-negative random variables it takes the simpler form

$$\lim_{x \to +\infty} \lim_{n \to +\infty} \inf F_n(x) = 1$$

The following properties will be useful in the sequel.

Property 1. A stable sequence is substable

Property 2. If X_n and Y_n are substable sequences, the sequence $(X_n + Y_n)$ is substable.

Property 3. Let Y_n, X_n, Z_n be sequences of proper random variables with distributions G_n, F_n, H_n, respectively. If Y_n, Z_n are substable and $G_n(x) \leq F_n(x) \leq H_n(x)$, for every x and n, then X_n is substable.

The interconnected system will be called stable, substable, or unstable if the induced packet delay process $\{\delta_\ell, \ell=1,2,\ldots\}$ is stable, substable, or unstable respectively. Similarly, a queue will be called stable, substable, or unstable according to the behavior of the corresponding queue size process.

Our main problem will be to try to find simple conditions on the input rates λ_1

and λ_2 that will guarantee (sub) stability of the interconnected system, or of a particular queue in the system. In particular, we will be interested in determining the _stability region_ of the system, which is defined as

$$S \stackrel{\Delta}{=} \{(\lambda_1, \lambda_2): \lim_{x \to +\infty} \lim_{\ell \to +\infty} \inf \Pr(\delta_\ell < x) = 1\} \tag{2}$$

(If the system is stable, then we will write "lim" instead of "lim inf")

D. The Dominant System

In this section we construct an auxiliary system, called _dominant system_, to be used in the analysis of the real system. Network i, queue i, and stack i of the dominant system will be referred to as dominant network i, dominant queue i, and dominant stack i, respectively; also, a quantity X defined in the real system will be denoted by $\bar{X}$ in the dominant system.

Except for the modifications in the transmission policies described below, the dominant system coincides with the real system in all other respects. In particular, the two systems have identical packet generation and routing patterns. That is, a packet $\bar{\xi}$ is generated at instant t_ξ in dominant network i if and only if a packet ξ is generated at instant t_ξ in network i of the real system (packet $\bar{\xi}$ can be thought of as the copy of packet ξ); also, packet $\bar{\xi}$ is an internetwork packet if and only if packet ξ is an internetwork packet; that is,

$$\bar{\rho}_{\bar{\xi}}^i \equiv \rho_\xi^i \quad , \text{ for every } \xi. \tag{3}$$

Transmission Policy for the Bridge Nodes in the Dominant System

If $\bar{Q}_n^i > 0$, then, as in the real system, bridge node i in the dominant system transmits a packet if and only if $\bar{T}_n^i = 1$, where

$$\bar{T}_n^i \equiv T_n^i \quad , \text{ for every } n. \tag{4}$$

Modification 1. If $\bar{Q}_n^i = 0$, then bridge node i in the dominant system transmits a _fictitious_ packet if and only if $\bar{T}_n^i = 1$. A fictitious packet is not included in the number of packets in the dominant queue, and it is removed from the dominant

queue upon arrival of an ordinary packet.

Transmission Policy for the Users in the Dominant System.

Except for the following modification, the transmission rules for local packets in the dominant system are a repetition of the rules defined for the real system, with F_n^i, ξ, $I_n^i(\xi)$, $U_n^i(\xi)$ replaced by $\bar{F}_n^i$, $\bar{\xi}$, $\bar{I}_n^i(\bar{\xi})$, $\bar{U}_n^i(\bar{\xi})$, respectively, where $\bar{U}_n^i(\bar{\xi}) \equiv U_n^i(\xi)$, for every n and ξ.

Modification 2. At the end of each slot n for which $\bar{F}_n^i = C$ and $F_n^i = NC$, the updating of the counter indication $\bar{I}_n^i(\bar{\xi})$ of a packet $\bar{\xi}$ in dominant stack i is done as follows:

a) If $\bar{I}_n^i(\bar{\xi}) = 0$, then $\bar{I}_{n+1}^i(\bar{\xi}) = J_{n+1}^i(\bar{\xi})$, where $J_{n+1}^i(\bar{\xi})$ is an integer random variable uniformly distributed on $\{\bar{K}_n^i - 1, \bar{K}_n^i, \ldots, \bar{K}_n^i + n - 2\}$, independent of any other variable in the system; where $\bar{K}_n^i$ is the position of the pointer in dominant stack i at time n. ($\bar{K}_n^i$ is updated according to the rules defined for K_n^i.)

b) If $\bar{I}_n^i(\bar{\xi}) > 0$, then $\bar{I}_{n+1}^i(\bar{\xi}) = \bar{I}_n^i(\bar{\xi}) - 1$.

The rationale for the particular construction of the dominant system defined above is the following. Modification 1 eliminates the dependency of a slot outcome in network i on the state of the queue of bridge node i ($Q_n^i = 0$ vs $Q_n^i > 0$), and, therefore, makes the dominant system easier to analyze. However, to be able to relate the dominant system to the real one, we need modification 2, it can be seen, after a little (or a lot of) thought, that the following property is true.

Property A: If at time n, n=1,2,..., packet ξ is in cell j, $(0 \leq j < K_n^i)$, of stack i, then, at time n, packet $\bar{\xi}$ —i.e., the copy of packet ξ— is in cell j of dominant stack i.

Property B: A packet ξ departs stack i either at the same time or before packet $\bar{\xi}$ departs dominant stack i.

The above properties are the key in proving the following proposition, which specifies in what sense the constructed auxiliary system dominates the real system.

Proposition 1

Let (Ω, F, P) be the common probability space of all random variables defined so far.

The following inequalities are true for every $\omega \in \Omega$ and every time n; n=1,2,.....

$$\bar{C}_n^i(j) \geq C_n^i(j) \; ; \; i=1,2; \; j=0,1,2,\ldots \qquad (5)$$

$$\bar{B}_n^i \geq B_n^i \; ; \; i=1,2 \qquad (6)$$

$$\bar{B}_n^1 + \bar{Q}_n^2 \geq B_n^1 + Q_n^2 \qquad (7.a)$$

$$\bar{B}_n^2 + \bar{Q}_n^1 \geq B_n^2 + Q_n^1 \qquad (7.b)$$

Proof: Inequalities (5), (6) follow directly from property A. To prove (7.a) consider the two-queue system $S_1 = (DQ^1, Q^2)$ in the real system (Figure 5), and the corresponding system $\bar{S}_1 = (D\bar{Q}^1, \bar{Q}^2)$ in the dominant system. Consider a packet ξ and its copy $\bar{\xi}$, and assume that $\bar{\xi}$ departs $\bar{S}_1$ during slot n. If $\bar{\xi}$ is not an internetwork packet, then, by (3) and property A, we have that packet ξ departed system S_1 during slot k, where $k \leq n$. If packet $\bar{\xi}$ is an internetwork packet, then it departs $\bar{S}_1$ from $\bar{Q}^2$. Define the variables Δ_j^2, and $\bar{\Delta}_j^2$ as follows:

$$\Delta_j^2(\bar{\Delta}_j^2) = \begin{cases} 1 \text{ if a packet departs } Q^2(\bar{Q}^2) \text{ during slot j} \\ 0 \text{ otherwise} \end{cases}$$

Note that $\Delta_j^2 = T_j^2 \, I(C_j^1(0) = 0) \, I(Q_j^1 > 0)$, and $\bar{\Delta}_j^2 = \bar{T}_j^2 \, I(\bar{C}_j^1(0)=0) \, I(\bar{Q}_j^2 > 0)$, where $I(\cdot)$ is the indicator function of the event in the parenthesis. If $Q_n^2 > 0$, then, by (4) and (5), the event $\bar{\Delta}_n^2 = 1$ implies the event $\Delta_n^2 = 1$. If $Q_n^2 = 0$, then by property B, packet ξ departed Q^2 during slot ℓ, where $\ell \leq n$. Thus, we have shown that a packet departure from system $\bar{S}_1$ either implies a simultaneous departure from S_1, or that the departing packet from $\bar{S}_1$ is a copy of a packet that has already departed S_1. The fact that both system S_1 and $\bar{S}_1$ have identical packet generation patterns completes the proof of (7.a). The proof of (7.b) is the same.

E. Stability Analysis of the Dominant System

In this section we establish the conditions for substability of the queues in the dominant system.

The Dominant Distributed Queue

We first consider $\overline{DQ}^i$, i.e., dominant stack i, and the queue-size process $\{\overline{B}_n^i, n \geq 1\}$ associated with it. In contrast to the real system, in the dominant system local transmissions are subject to bridge node interference represented by the i.i.d. process $\{\overline{T}_n^i, n \geq 1\}$ which is __independent__ of any other process in the system, (modification 1). This property permits us to study the stability of $\overline{DQ}^i$ independently of any other component in the interconnected system.

Figure 6 shows $\overline{DQ}^i$ with its associated input $\overline{M}_n^i$, queue size $\overline{B}_n^i$, bridge node interference $\overline{T}_n^i$, and output $\overline{0}_n^i$ variables during slot n. Note that, by construction, $\overline{M}_n^i \equiv M_n^i$ for every n.

The output $\overline{0}_n^i$ is defined as follows:

$$\overline{0}_n^i = \begin{cases} 1 & \text{if slot n is busy with the successful user transmission.} \\ 0 & \text{otherwise} \end{cases}$$

Note that $\overline{0}_n^i = (1-\overline{T}_n^i) \, I(\overline{C}_n^i(0) = 1)$, where $I(\cdot)$ is the indicator function of the event in the parenthesis.

The subsystem of Figure 6 operates with the modified SA_n in __sessions__. The sessions are non-overlapping time intervals which partition the slotted time axis of channel i, and are defined as follows. Let

$$\overline{R}_1^i = 1 \; ; \; \overline{R}_{k+1}^i = \min \{m : n > \overline{R}_k^i, \; \overline{K}_n^i = 1, \; \overline{F}_{n-1}^i = NC\} \; , \; k \geq 1 \qquad (8)$$

The interval $[\overline{R}_k^i, \overline{R}_{k+1}^i)$ defines the k-th session of channel i in the dominant system. From (8) and the rules of the modified SA_n and its associated pointer, it is not difficult to see that at the instant just before the beginning of each session dominant stack i is empty of packets. This observation leads to the following properties:

Property 1: Session lengths are i.i.d. random variables. This follows from the fact that $\{\bar{M}_n^i; \, n \geq 0\}$ and $\{\bar{T}_n^i; \, n \geq 1\}$ are i.i.d processes independent of each other and of any other process in the system.

Remark: At first glance, it might seem that session lengths are dependent since the operation of the modified SA_n depends on the state of the real system; (see modification 2). However, this is not so, because modification 2 introduces, in effect, a mere reordering of cells, which does not affect the independence of session lengths.

Property 2: The user packets that were successfully transmitted during the k-th session of channel i, are the packets that were generated during the interval $[\bar{R}_k^i - 1, \, \bar{R}_{k+1}^i - 1)$.

Let us now define the following random variables.

$\bar{L}_k^i$: the session length of the k-th session in channel i

$\bar{G}_k^i$: the number of successful user transmissions during the k-th session of channel i.

Let, also

$$\lambda_i^* \overset{\Delta}{=} \sup \, \{\lambda_i : \, E(\bar{L}_1^i) < \infty\} \qquad (9)$$

where $\lambda_i = E(\bar{M}_1^i)$.

By definition we have that $\bar{L}_k^i = \bar{R}_{k+1}^i - \bar{R}_k^i$, and

$$\bar{R}_1^i = 1 \quad ; \quad \bar{R}_k^i = \bar{R}_1^i + \sum_{j=1}^{k-1} \bar{L}_j^i$$

By property 1, $\{\bar{R}_k^i; \, k \geq 1\}$ is a **renewal** process. We can now express the following proposition regarding the stability of $\overline{DQ}^i$.

Proposition 2. The process $\{\bar{B}_n^i; \, n \geq 1\}$ is substable if $\lambda_i < \lambda_i^*$.

Proof: Let $\{\bar{N}_n^i; \, n \geq 1\}$ be the counting renewal process associated with the renewal process $\{\bar{R}_k^i; \, k \geq 1\}$; that is,

$$\bar{N}_n^i = \max \, \{k : \, \bar{R}_k^i \leq n\}$$

Let also

$$\Lambda_n^i = \bar{G}_{\bar{N}_n^i}^i \quad , \, n \geq 1$$

By definition, the process $\{\Lambda_n^1; \; n{\geq}1\}$ is <u>regenerative</u>[2] with respect to the renewal process $\{\bar{R}_k^1; \; k{\geq}1\}$. Since for $\lambda_1 < \lambda_1^*$ we have that $E(\bar{L}_k^1){<}\infty$, it follows from the regenerative theorem [7, Thm.2] that $\{\Lambda_n^1; \; n{\geq}1\}$ is stable. Clearly, $0{\leq}\bar{B}_n^1{\leq}\Lambda_n^1$, for every $n{\geq}1$; thus, by property 3 of section II.C $\{\bar{B}_n^1; n{\geq}1\}$ is a substable process.

By property 2, we have that

$$\bar{G}_k^1 = \sum_{j=1}^{\bar{L}_k^1} \Xi_j \quad , \; k=1,2,\ldots \tag{10}$$

Where Ξ_j denotes the number of packets generated in slot $\bar{R}_k+j-2$. Note that $\bar{L}_k^1$ is a <u>stopping time</u> for $\{\Xi_j; \; j{\geq}1\}$, since the event $\{\bar{L}_k^1{=}m\}$ is independent of $\Xi_{m+1}, \; \Xi_{m+2}, \ldots$. If $E(\bar{L}_k^1){<}\infty$, or equivalently if $\lambda_1 < \lambda_1^*$, then, taking expectations in (10), and using Wald's theorem yields

$$E(\bar{G}_k^1) = \lambda_1 \; E(\bar{L}_k^1) \; , \; k=1,2,\ldots \tag{11}$$

The number of successful local transmissions, $\bar{G}_k^1$, during the k-th session can be thought of as a reward earned during the session. Note that $\bar{G}_k^1$ depends on $\bar{L}_k^1$, but the pairs $(\bar{L}_k^1, \; \bar{G}_k^1)$, $k{\geq}1$, are i.i.d.. Thus, by a well-known result from the theory of renewal reward processes [9, Thm.3.6.1] we have that if $\lambda_1 < \lambda_1^*$, then

$$\lim_{n\to+\infty} n^{-1} \sum_{j=1}^{n} \bar{O}_j^1 = \lim_{n\to+\infty} n^{-1} E(\sum_{j=1}^{n}\bar{O}_j^1) = \frac{E(\bar{G}_1^1)}{E(\bar{L}_1^1)} = \lambda_1 \qquad \text{a.e.} \quad (12)$$

, where the last equality in (12) follows from (11).

The Dominant Queue

Let us now consider the queue $\bar{Q}^1$ of bridge node 1 in the dominant system. In reference to Figure 7, the arrival process $\{\bar{A}_n^{1'}, \; n{\geq}1\}$ is the process of successful

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2. A discrete-time process $\{X_n; n{\geq}1\}$ is said to be regenerative with respect to the renewal process $\{R_k; \; k{\geq}1\}$, if for every positive integer M and every sequence $i_1,\ldots, \; i_m$, with $0{<}i_1{<}\ldots{<}i_m$, the joint distribution of $X_{i_1+R_k}, \ldots, X_{i_m+R_k}$ is independent of k.

internetwork packet transmissions in network i'; that is, $\bar{A}_n^{1'} =1$, if a user packet $\bar{\xi}$ is successfully transmitted in network i' during slot n, (i.e., $\bar{0}_n^{1'} =1$), and packet $\bar{\xi}$ is an internetwork packet, (i.e., $\bar{\rho}_{\bar{\xi}}^{1'} = 1$); $\bar{A}_n^{1'} =0$, otherwise. The departure process $\{\bar{D}_n^1; n\geq 1\}$ is defined as follows:

$$\bar{D}_n^1 = \begin{cases} 1 & \text{if dominant bridge node i successfully transmits a (real or fictitious) packet in slot n} \\ 0 & \text{otherwise} \end{cases}$$

Recall that in the dominant system bridge node i transmits a (real or fictitious packet in slot n if $\bar{T}_n^1 =1$, and that its transmission is successful if no user packet is transmitted in slot n, that is, if $\bar{C}_n^1(0) = 0$. Thus,

$$\bar{D}_n^1 = \bar{T}_n^1 \cdot I(\bar{C}_n^1(0) =0)$$

In the stability analysis of $\bar{Q}^1$ we will make use of the fact that both channel i and channel i' operate with the modified SA_n in sessions of i.i.d. lengths. To this end, let us define the following random variables.

$\bar{S}_k^1$: the number of user internetwork packets that were successfully transmitted during the k-th session of channel i.

$\bar{H}_k^1$: the number of (real or fictitious) node packets that were successfully transmitted during the k-th session of channel i.

Note that, by construction of the dominant system and the session, the quadruples $(\bar{L}_k^1, \bar{G}_k^1, \bar{S}_k^1, \bar{H}_k^1)$, $k\geq 1$, are i.i.d., and the sequence $(\bar{L}_k^1, \bar{G}_k^1, \bar{S}_k^1, \bar{H}_k^1)$, $k\geq 1$, is independent of the sequence $(\bar{L}_k^2, \bar{G}_k^2, \bar{S}_k^2, \bar{H}_k^2)$, $k\geq 1$.

Consider channel i and assume that $\lambda_i < \lambda_i^*$. From (3) and (11) we have that

$$E(\bar{S}_k^1) = p_i \lambda_i E(\bar{L}_k^1), \quad k=1,2,\ldots \qquad (13)$$

Also, by theorem 2 in [7], as in (10), we have that

$$\lim_{n \to +\infty} n^{-1} \sum_{j=1}^{n} \bar{A}_j^i = \lim_{n \to +\infty} n^{-1} E\left(\sum_{j=1}^{n} \bar{A}_j^i\right) = \frac{E(\bar{S}_1^i)}{E(\bar{L}_1^i)} = p_i \lambda_i \qquad \text{a.e.} \qquad (14)$$

$$\lim_{n \to +\infty} n^{-1} \sum_{j=1}^{n} \bar{D}_j^i = \lim_{n \to +\infty} n^{-1} E\left(\sum_{j=1}^{n} \bar{D}_j^i\right) = \frac{E(\bar{H}_1^i)}{E(\bar{L}_1^i)} \qquad \text{a.e.} \qquad (15)$$

where the last equality in (14) follows from (13). Let us define

$$d_i \triangleq \frac{E(\bar{H}_1^i)}{E(\bar{L}_1^i)} \qquad (16)$$

We can now express the following theorem regarding the substability of $\bar{Q}^i$

Theorem 1

The process $\{\bar{Q}_n^i; n \geq 1\}$ is substable if

$$\lambda_i < \lambda_i^* \ , \ \lambda_{i'} < \lambda_{i'}^* \ , \ p_i \lambda_{i'} < d_i \qquad (17)$$

The proof of theorem 1 can be found in the Appendix.

Remarks:

1) In view of (14), (15), (16), and proposition 1, theorem 1 states that $\bar{Q}^i$ is substable if both $\mathcal{D}\bar{Q}^i$ and $\mathcal{D}\bar{Q}^{i'}$ are substable and the (expected) long-run average number of arrivals $(p_i \lambda_{i'})$ is less than the (expected) long-run average number of departures (d_i).

2) The stability of queues with non-i.i.d. arrival and departure processes has been studied by Loynes [5], (see also [6]). These studies assume strict stationarity of the arrival and departure processes. In our case, however, we cannot claim stationarity or even asymptotic stationarity of the processes $\{\bar{A}_n^{i'}; n \geq 1\}$ and $\{\bar{D}_n^i; n \geq 1\}$, since the operation of the dominant system depends on the operation of the underlying real system. Nevertheless, theorem 1 shows that simple "intuitive" stability criteria can still be derived when the arrival and departure processes are associated with renewal processes.

F. Stability of the Real System.

In this section we combine the stability results for the dominant system with the dominance relation, as expressed by proposition 1, to derive sufficient conditions for the stability of the real system. We begin with the following theorems

Theorem 2

The processes $\{B_n^1; n \geq 1\}$, $\{B_n^2; n \geq 1\}$, $\{Q_n^1; n \geq 1\}$, and $\{Q_n^2; n \geq 1\}$ are substable if

$$\lambda_1 < \lambda_1^*, \quad \lambda_2 < \lambda_2^*, \quad p_1 \lambda_1 < d_2, \quad p_2 \lambda_2 < d_1 \qquad (18)$$

Proof:

By proposition 2 and theorem 1, processes $\{\bar{B}_n^i; n \geq 1\}$ and $\{\bar{Q}_n^i; n \geq 1\}$, $i=1,2$, are substable. Thus, from property 2 of Section II.C we have that the processes $\{\bar{B}_n^i + \bar{Q}_n^{i'}, n \geq 1\}$, $i=1,2$, are substable. From proposition 1 we have that, for every $\omega \in \Omega$ and every n, $n \geq 1$,

$$0 \leq B_n^i \leq \bar{B}_n^i, \quad i=1,2,$$

$$0 \leq Q_n^i \leq Q_n^i + B_n^{i'} \leq \bar{Q}_n^i + \bar{B}_n^{i'}, \quad i=1,2,$$

The theorem follows from property 3 of Section II.C.

Theorem 2 has the following interesting corollary, whose proof can be found in the Appendix.

Theorem 3

If (18) holds, then the processes $\{B_n^1, n \geq 1\}, \{B_n^2, n \geq 1\}, \{Q_n^1, n \geq 1\}, \{Q_n^2, n \geq 1\}$ are stable, and the Markov chain $\{Z_n; n \geq 0\}$, defined in (1), is ergodic.

Theorem 4

If (18) holds, then the system is stable; that is,

$$\lim_{x \to \infty} \lim_{\ell \to \infty} Pr(\delta_\ell < x) = 1$$

where δ_ℓ is the delay of the ℓ-th packet.

Proof

Consider the Markov chain $\{Z_n; n \geq 0\}$, and let $\tau_1, \tau_2 \ldots$ be the times of successive visits to state $\underline{0}$, where $\underline{0}$ is the element $[0,0,(0),(0),1,1]$ of the chain's state space. (Note that $Z_0 = \underline{0}$, by definition). By theorem 2, $\{Z_n; n \geq 0\}$ is ergodic; therefore, the process $\{0, \tau_1, \tau_2, \ldots\}$ is a renewal process with finite interrenewal time, that is, $E(\tau_{i+1} - \tau_i) = E(\tau_1) < \infty$, $i = 1, 2, \ldots$. Consider, now, the packet delay process $\{\delta_\ell; \ell \geq 1\}$ and observe that it is regenerative with respect to $\{0, \tau_1, \tau_2, \ldots\}$. Since $E(\tau_1) < \infty$, it follows from [7, Thm. 2] that $\{\delta_\ell; \ell \geq 1$ is stable.

Let $S \overset{\Delta}{=} \{(\lambda_1, \lambda_2) : \lambda_1 < \lambda_1^*, \lambda_2 < \lambda_2^*, p_1 \lambda_1 < d_2, p_2 \lambda_2 < d_1\}$. Clearly, S is a subset of the system stability region S, as defined in (2). To determine S we need to compute the quantities $\lambda_1^*, \lambda_2^*, d_1$, and d_2, which, by definition, are functions of the system parameters $\lambda_1, \lambda_2, \pi_1, \pi_2$, and n; in particular $\lambda_i^* = \lambda_i^*(n, \pi_i)$ and $d_i = d_i(n, \pi_i, \lambda_i)$. Given the system parameters, λ_i^* and d_i are determined using only quantities that refer to one session of the modified SA_n; specifically, the expected session length, $E(\overline{L}_1^{-1})$, and the expected number of successful node-transmissions during a session, $E(\overline{H}_1^{-1})$. Since both the $E(\overline{L}_1^{-1})$ and the $E(\overline{H}_1^{-1})$ are not affected by modification 2 of Section II.B, the analysis methods used in [2,3,4] for the original SA_n can be used to compute λ_i^* and d_i. Figure 8 shows λ_i^* as a function of π_i for $n=2$ and $n=3$. Note that, for $\pi_i = 0$, λ_i^* is the throughput of the SA_n, [3]. Figure 9 gives the plots of d_i versus λ_i for $n=2,3$ and $\pi_i = 0.25, 0.5$, and 0.75. The stability subregion S, as determined by the obtained values of λ_i^* and d_i, is shown in Figures 10, and 11 for $n=2,3$, and for different values of π_1, π_2, p_1, and p_2.

III. CHANNEL DIVISION MULTIPLEXING

Under channel division multiplexing, the available broadcast channel in each of the two networks is divided into a note subchannel dedicated for node transmissions and a random-access user subchannel dedicated for user transmissions. The channel

The channel division may be done either in the frequency or in the time domain.

Under frequency division multiplexing (FDM), the available bandwidth W of channel i is partitioned into a frequency band of width $\alpha_N^i W$, assigned to the node subchannel, and a frequency band of width $\alpha_U^i W$, assigned to the user subchannel;where

$$\alpha_N^i \geq 0 \ , \ \alpha_U^i \geq 0 \ , \ \alpha_N^i + \alpha_U^i = 1 \qquad (19)$$

As long as its queue is non-empty, bridge node i transmits a packet (with probability one) over the node subchannel. Since there is no multi-access interference and the channel is assumed errorless, node transmissions are always successful. However, the packet transmission time and, therefore, the service time of Q^i, is now $1/\alpha_N^i$ units of time. The random-access user subchannel is shared by the users in accordance with the SA_n described in Section II.A; the packet transmission time, and, therefore, the channel's slot size is now $1/\alpha_U^i$ units of time.

Under time division multiplexing (TDM), time is divided into successive periods called <u>frames.</u> Each frame contains M^i unit time successive slots, K^i of which are assigned to bridge node i and $M^i - K^i$ are assigned to the local users; where $K^i/M^i = \alpha_N^i$, $1-K^i/M^i = \alpha_U^i$, and $0 \leq K^i \leq M^i$. The distribution of the K^i node slots and the $M^i - K^i$ user slots over the time frame can be arbitrary, but fixed. If its queue is non-empty, bridge node i transmits a packet (with probability one) in the first available node slot. Successive user slots form a interleaved random-access channel which is shared by the local users in accordance with the SA_n.

The stability analysis of the interconnected system, under either FDM or TDM, is simplified by the fact that the two tandem subsystems $S^1 = (DQ^1, Q^2)$ and $S^2 = (DQ^2, Q^1)$ evolve independently in time, once the fractions α_N^i, α_U^i, i=1,2, have been fixed.

The stability of system S^1 depends on whether both its constituent queues are stable or not. Consider first DQ^1. From [2,3], we have that the SA_n, induces finite expected packet delays if and only if $\lambda_i < \bar{\lambda}_i$, where $\bar{\lambda}_i$ is the <u>throughput</u> of the SA_n

in packets per unit time; ($\bar{\lambda}_1 = 0.360$ for n=2, $\bar{\lambda}_1 = 0.401$ for n=3). In our case, the random-access user subchannel occupies a fraction α_U^i of the overall channel; thus, the SA_n throughput is $\alpha_U^1 \bar{\lambda}_1$ and, therefore, $\mathcal{DQ}^1$ is stable if

$$\lambda_1 < \alpha_U^1 \cdot \bar{\lambda}_1 \tag{20}$$

If $\mathcal{DQ}^1$ is stable, then the arrival rate to $Q^{1'}$ is $p_1 \lambda_1$; also, the service time of $Q^{1'}$ is equal to $1/\alpha_N^{1'}$. For the stability of $Q^{1'}$ it will be required that

$$p_1 \lambda_1 < \alpha_N^{1'} \tag{21}$$

The following theorem combines (20) with (21) to give a sufficient condition for system stability.

Theorem 5

The system is stable if

$$\lambda_1 < \alpha_U^1 \bar{\lambda}_1 \ , \ \lambda_2 < \alpha_U^2 \bar{\lambda}_2 \ , \ p_1 \lambda_1 < \alpha_N^2 \ , \ p_2 \lambda_2 < \alpha_N^1 \tag{22}$$

The proof of theorem 5 is based on the same ideas used in the proofs of theorem 1 and 3, and, therefore, it is omitted.

In contrast to contention multiplexing, for channel division multiplexing the stability subregion $\bar{S} \overset{\Delta}{=} \{(\lambda_1, \lambda_2): (22) \text{ is satisfied }\}$ is always a rectangle in the (λ_1, λ_2) plane. Given $\lambda_1, \lambda_2, p_1, p_2$, and the algorithm parameter n, the channel division parameters α_U^i, α_N^i, i=1,2, should be chosen so that (19) and (22) are satisfied (if possible). The best choice for n is $n=3$, since it results in the highest SA_n throughput ($\bar{\lambda}_1 = 0.401$), and uniformly better expected packet delay characteristics [2]. The choice of α_U^i, α_N^i depends on the traffic parameters $\lambda_1, \lambda_2, p_1$ and p_2. For example, in the symmetric case, where $\lambda_1 = \lambda_2 = \lambda, p_1 = p_2 = p, \bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$, the largest set of λ's satisfying (22) is obtained if we choose

$$\alpha_N^1 = \alpha_N^2 = \frac{p\bar{\lambda}}{p\bar{\lambda}+1} \quad , \quad \alpha_U^1 = \alpha_U^2 = \frac{1}{p\bar{\lambda}+1} \qquad (23)$$

In this case, the system is stable if

$$\lambda < \frac{\bar{\lambda}}{p\bar{\lambda}+1}$$

where $\bar{\lambda}=0.401$, and $0 \leq p \leq 1$.

IV. CONCLUDING REMARKS

In this paper we have given sufficient conditions for the stability of a system of interacting queues that models the interconnection of two random-access broadcast channels. The stability analysis of the system with contention multiplexing has been based on the stability analysis of a dominant system, which is analytically more tractable than the original system and its stability guarantees the stability of the original system. The dominant system technique is quite useful in studying the stability of systems with multiple interacting queues and has been used in several recent studies [8,10,11,12]. In studying the stability of the bridge node queues, we have shown that simple intuitive stability criteria can be rigorously established when the arrival and departure processes are associated with renewal processes.

The system presented here may be extended to include more than two networks, and it may be modified to operate under different RAA's and channel models, (e.g., carrier sensing). The analysis of this paper provides a framework for the study of such extension and modifications.

Finally, we note that, since the derived stability conditions are only sufficient, we have avoided making performance comparisons between the system with contention multiplexing and the system with channel division multiplexing. In general, however, the obtained stability subregions for the two systems are comparable. Which multiplexing technique is best for given traffic requirements and delay constraints can be determined only if we knew the packet delay distributions. One step towards this direction would be the determination of the first few moments of the packet delay; the delay analysis method developed in [14] could be useful in this respect.

APPENDIX

PROOF OF THEOREM 1

We will present the proof for i=1, the proof for i=2 is the same.

The difficulty in proving theorem 1 lies in the fact that the operation of the dominant system depends on the original system, and, therefore, we cannot claim stationarity or even asymptotic stationarity of the processes $\{\bar{A}_n^1 ; n \geq 1\}$, $\{\bar{D}_n^2 ; n \geq 1\}$. Nevertheless, the following equalities are true:

$$\bar{Q}_{n+1}^2 = \max (\bar{Q}_n^2 - \bar{D}_n^2 , 0) + \bar{A}_n^1 = \max (\bar{Q}_n^2, \bar{D}_n^2) + \bar{C}_n \qquad (A.1)$$

where $\bar{C}_n = \bar{A}_n^1 - \bar{D}_n^2$

It can be shown by induction, from (A.1), that

$$\bar{Q}_{n+1}^2 = \max (c_{1n} + \bar{Q}_1^2, c_{2n} + \bar{A}_1^1, \ldots , c_{mn} + \bar{A}_{m-1}^1, \ldots, \bar{A}_n^1) \qquad (A.2)$$

where $c_{nm} = \sum_{j=m}^{n} \bar{C}_j$

By definition, the following equalities hold [3]

$$\bar{S}_{\bar{N}_n^1}^1 = \sum_{n=\bar{N}_n^1}^{u_n^1} \bar{A}_n^1 \quad , \quad \bar{H}_{\bar{N}_n^2}^2 = -\sum_{n=\bar{N}_n^2}^{u_n^2} \bar{D}_n^2 \qquad (A.3)$$

where $u_n^i = \bar{N}_n^i + \bar{L}_{\bar{N}_n^i}^i$, i=1,2, and $\bar{N}_n^i$ is the counting renewal process defined in the proof of proposition 2.

From (A.3) we conclude that

$$\sum_{j=m}^{n} \bar{A}_j^1 \leq \sum_{k=\bar{N}_m^1}^{\bar{N}_n^1} \bar{S}_k^1 \quad , \quad \sum_{j=m}^{n} \bar{D}_j^2 \geq \sum_{k=\bar{N}_m^2+1}^{\bar{N}_n^2-1} \bar{H}_k^2 \qquad (A.4)$$

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3. In formulae (A.3), (A.4) we adopt the notation

$$\sum_{k=i}^{j} \alpha_k = 0 \qquad \text{if } i > j$$

From (A.2), (A.4), and the fact that $\bar{A}_n^1 \leq 1$, and $\bar{Q}_1^2 \geq 1$, we obtain

$$\bar{Q}_{n+1}^2 \leq \phi_n + \bar{Q}_1^2 + 1 \quad , \quad n \geq 1 \quad\quad (A.5)$$

where

$$\phi_n = \max \left(\max_{1 \leq m \leq n} \left(\sum_{k=\bar{N}_m^1}^{\bar{N}_n^1} \bar{S}_k^1 - \sum_{k=\bar{N}_m^2+1}^{\bar{N}_n^2+1} \bar{H}_k^2 \right), 0 \right)$$

We will show that the process $\{\phi_n; n \geq 1\}$ is stable. The substability of $\{\bar{Q}_n^2; n \geq 1\}$ will then follow from (A.5), and properties 1,2, and 3 of Section II.C, since $\bar{Q}_1^2$ is a proper random variable; (in fact, if we assume that the system is initially empty, $\bar{Q}_1^2 = 0$).

To prove the stability of $\{\phi_n; n \geq 1\}$ we need some preliminary results. First, a strengthening of Theorem 4.3 in [15]. We will consider the discrete case, since this is of interest to us.

Let $(x_k(n,\omega), X_k(\omega))$, $k = \pm 1, \pm 2, \pm 3, \ldots$ be i.i.d. random pairs defined on some probability space (Ω, F, P). For every $k, X_k(\cdot)$, is nonnegative, proper and integer valued, while $x_k(n, \cdot)$ is a sequence of proper random variables (or vectors). Define:

$$S_1 = 0$$

$$S_k = \sum_{i=1}^{k-1} X_i \quad ; \quad k \geq 2$$

$$N_n = \max(k: \; S_k \leq n) = \min(k: \; \sum_{i=1}^{k} X_i > n) \quad ; \quad n \geq 1$$

$$z_n = n - S_{N_n} \quad ; \quad n \geq 1$$

$$v_n = x_{N_n}(z_n) \quad ; \quad n \geq 1$$

Note that, by definition, the process $\{v_n; n \geq 1\}$ is regenerative with respect to the renewal process $\{S_k; k \geq 1\}$. z_n is called the "current life" of the process $\{X_k\}$.

Let $((x_0(n,\omega), X_0(\omega)), z_0^*(\omega))$ be a random triplet defined on (Ω, F, P), which is independent of $(x_k(n,\omega), X_k(\omega))$, $k = \pm 1, \pm 2, \pm 3, \ldots$, and define

$$S_k^* = -z_0^* + \sum_{i=0}^{k-1} X_i \quad ; \quad k>0$$

$$S_0^* = -z_0^* \quad\quad\quad ; \quad k=0$$

$$S_k^* = -z_0^* - \sum_{i=-1}^{k} X_i \quad ; \quad k>0$$

$$N_n^* = \max(k: S_k^* \leq n) \quad ; \quad n\epsilon Z$$

$$z_n^* = n - S_{N_n}^* \quad\quad\quad ; \quad n\epsilon Z$$

$$v_n^* = x_{N_n^*}(z_n^*)$$

Let $E(X_1) < \infty$, and let the distribution of X_1 be aperiodic. Then, if the triplet $((x_0(n,\omega), X_0(\omega)), z_0^*(\omega))$ is appropriately constructed, it is shown in [15] that the resulting process $\{v_n^* ; n\epsilon Z\}$ is strictly stationary and that

$$\lim_{n\to\infty} P(v_{n+k_i} \leq a_i \quad ; \quad i=1,\ldots,m) = P(v_{k_i}^* \leq a_i \quad ; \quad i=1,\ldots,m) \quad ; \quad k_i \epsilon Z$$

Let now, $\{v_n\}$ be the sequence $\{v_n, v_{n+1}, \ldots, v_{n+k}, \ldots\}$ and $\{v_0^*\}$ be the sequence $\{v_0^*, v_1^*, \ldots, v_k^*, \ldots\}$. We will show that

$$\lim_{n\to\infty} P(\{v_n\} \epsilon B^\infty) = P(\{v_0^*\} \epsilon B^\infty)$$

uniformly over all measurable subsets B^∞ of R^∞, where R^∞ is the space of all two-sided real sequences.

We first need the following lemma, which is a special case of Scheffe's Theorem [13, Appendix II].

Lemma A.1

Let $p_{ni} \geq 0$, $\pi_i \geq 0$, $\sum_{i=1}^{\infty} p_{ni} = \sum_{i=1}^{\infty} \pi_i = 1$, $\lim_{n\to\infty} p_{ni} = \pi_i$; $i \geq 0$, $n \geq 1$. Then

$$\lim_{n\to\infty} \sum_{i=0}^{\infty} | p_{ni} - \pi_i | = 0$$

The above lemma has the following corollary:

Corollary A.1

Let the conditions of Lemma A.1 hold. Let N be a set of indices, and for $\nu \in N$, let c_i^ν, $i > 0$, be a sequence of reals, such that $|c_i^\nu| < \Gamma < +\infty$, for every $i \geq 0$ and $\nu \in N$. Then,

$$\lim_{n \to +\infty} \sum_{i=0}^{\infty} c_i^\nu \, p_{ni} = \sum_{i=0}^{\infty} c_i^\nu \, \pi_i \quad , \text{ uniformly in } \nu$$

Proof: By Lemma A.1, we have

$$\left| \sum_{i=0}^{\infty} c_i^\nu \, p_{ni} - \sum_{i=0}^{\infty} c_i^\nu \, \pi_i \right| \leq \Gamma \sum_{i=0}^{\infty} |p_{ni} - \pi_i| \xrightarrow[n \to \infty]{} 0, \text{ uniformly in } \nu.$$

We can now express the following theorem:

Theorem A.1

Let $E(X_1) < \infty$, and X_1 be aperiodic. Then,

$$\lim_{n \to \infty} P(\{v_n\} \in B^\infty) = P(\{v_0^*\} \in B^\infty)$$

uniformly over all measurable subsets B^∞ of R^∞

Proof: As in [15],

$$P(\{v_n\} \in B^\infty \mid z_n = i) = P(\{v_0^*\} \in B^\infty \mid z_0^* = i)$$

Also, because of aperiodicity,

$$P(z_n = i) \xrightarrow[n \to \infty]{} P(z_0^* = i)$$

By the total probability theorem we have

$$P(\{v_n\} \in B^\infty) = \sum_{i=0}^{\infty} P(\{v_n\} \in B^\infty \mid z_n = i) \, P(z_n = i)$$

$$P(\{v_0^*\} \in B^\infty) = \sum_{i=0}^{\infty} P(\{v_0^*\} \in B^\infty \mid z_0^* = i) \, P(z_0^* = i)$$

The theorem now follows from corollary A.1, by identifying P_{ni}^{ν} with $P(z_n=i)$, π_i with $P(z_0^*=i)$, and c_i^{ν} with $P(\{v_0^*\} \in B^{\infty}|z_0^*=i)$; (the index set N corresponds to the Borel sigma field associated with R^{∞}).

In the proof of theorem 2, we will also need the following Lemmas.

<u>Lemma A.2</u>

Let (Ω,F) , (Ω',F') be measurable spaces, and let X_n, X^*, Y_n, Y^* be measurable mappings from Ω to Ω'. Let P be a probability measure on (Ω,F). If

a) $\{X_n;n\geq 1\}$ is independent of $\{Y_n;n\geq 1\}$, and X^* is independent of Y^*

b) $\lim\limits_{n\to+\infty} P(X_n \in B) = P(X^* \in B)$, uniformly in $B \in F'$

c) $\lim\limits_{n\to+\infty} P(Y_n \in B) = P(Y^* \in B)$, uniformly in $B \in F'$

then, $\lim\limits_{n\to+\infty} P((X_n, Y_n) \in C) = P((X^*, Y^*) \in C)$, unformly over all measurable sets C

of the product space $(\Omega'\times \Omega', F'\times F')$.

<u>Proof:</u> Let $B\in F'$, and define $P_{n1}(B) = P(X_n \in B), P_{n2}(B) = P(Y_n \in B)$,

$P_1^*(B) = P(X^* \in B)$, $P_2^*(B) = P(Y^* \in B)$; P_{n1}, P_{n2}, P_1^*, P_2^* are probability measures on (Ω',F').

Let $C \in F'\times F'$, and P_{no}, P_o^* be the product measures on $(\Omega'\times\Omega', F'\times F')$ induced by (P_{n1}, P_{n2}) and (P_1^*, P_2^*), respectively. Let also

$$C(\omega_1) = \{\omega_2 \in \Omega' : (\omega_1, \omega_2) \in C\}$$

$$C(\omega_2) = \{\omega_1 \in \Omega' : (\omega_1, \omega_2) \in C\}$$

Then, by Fubini's theorem, we have

$$P_{no}((X_n,Y_n) \in C) = P_{no}(C) = \int_{\Omega'} P_{n2}(C(\omega_1)) P_{n1}(d\omega_1) =$$

$$= \int_{\Omega'} (P_{n2}(C(\omega_1)) - P_2^*(C(\omega_1))) P_{n1}(d\omega_1) + \int_{\Omega'} P_2^*(C(\omega_1)) P_{n1}(d\omega_1) \qquad (A.6)$$

$$P((X^*, Y^*) \in C) = P_o^*(C) = \int_{\Omega'} P_1^*(C(\omega_2)) \; P_2^*(d\omega_2) =$$

$$= \int_{\Omega'} (P_1^*(C(\omega_2)) - P_{n1}(C(\omega_2))) \; P_2^*(d\omega_2) + \int_{\Omega'} P_{n1}(C(\omega_2)) \; P_2^*(d\omega_2) \qquad (A.7)$$

Also,
$$\int_{\Omega'} P_{n1}(C(\omega_2)) \quad P_2^*(d\omega_2) = \int_{\Omega'} P_2^*(C(\omega_1)) \; P_{n1}(d\omega_1) \qquad (A.8)$$

From (A.6), (A.7), and (A.8) we obtain

$$| P((X_n, Y_n) \in C) - P((X_*^*, Y^*) \in C)| =$$

$$= |\int_{\Omega'} (P_{n2}(C(\omega_1)) - P_2^*(C(\omega_1))) \; P_{n1}(d\omega_1) -$$

$$- \int_{\Omega'} (P_1^*(C(\omega_2)) - P_{n1}(C(\omega_2))) \; P_2^*(d\omega_2)|$$

$$\leq \int_{\Omega'} |P(Y_n \in C(\omega_1)) - P(Y^* \in C(\omega_1))| \; P_{n1}(d\omega_1) +$$

$$+ \int_{\Omega'} |P(X_n \in C(\omega_2)) - P(X^* \in C(\omega_2))| \; P_2^*(d\omega_2) \qquad (A.9)$$

By hypothesis, for every $\epsilon > 0$ there exists $N(\epsilon)$, such that

$$|P(X_n \in C(\omega_2)) - P(X^* \in C(\omega_2))| < \epsilon/2 \qquad (A.10)$$

for every $C(\omega_2)$ and $n \geq N(\epsilon)$.

$$P(Y_n \in C(\omega_1)) - P(Y^* \in C(\omega_1))| > \epsilon/2 \qquad (A.11)$$

for every $C(\omega_1)$ and $n \geq N(\epsilon)$.

From (A.9), (A.10), and (A.11), we obtain

$$| P((X_n, Y_n) \in C) - P((X^*, Y^*) \in C)| \leq \epsilon$$

for every $C \in F' \times F'$ and $n \geq N(\epsilon)$, and the proof is complete.

Lemma A.3

Let Y_m^n be a random variable defined on a probability space (Ω, F, P) for each $n > 0$ and $m \leq n$. If

$$\limsup_{n \to +\infty} Y_m^n < 0 \quad , \text{ for every } m, \tag{A.12}$$

then, for every N, the following are true:

a) For every measurable set B of the induced probability space

$$\liminf_{n \to +\infty} P(\max_{1 \leq m \leq n}(\max(Y_m^n), 0) \in B) = \liminf_{n \to +\infty} P(\max_{N \leq m \leq n}(\max(Y_m^n), 0) \in B)$$

b) The same equality holds with "lim inf" replaced by "lim sup".

Proof: Let

$$\phi_n^i = \max_{i \leq m \leq n}(\max(Y_m^n), 0) \quad ; \quad i \leq n$$

By (A.12), for each $\omega \in \Omega$, there exists an integer $M(\omega)$, such that $Y_m^n(\omega) < 0$, for every n and m, such that $n \geq M(\omega)$ and $1 \leq m < N$. Therefore,

$$\phi_n^1(\omega) = \phi_n^N(\omega), \quad n \geq M(\omega) \tag{A.13}$$

Since $M(\omega)$ is a proper random variable, using (A.13) we have

$$P(\phi_n^1 \in B) = \sum_{k=1}^{n} P(\phi_n^N \in B, M=k) + \sum_{k=n+1}^{+\infty} P(\phi_n^1 \in B, M=k)$$

or

$$P(\phi_n^1 \in B) = P(\phi_n^N \in B) - \sum_{k=n+1}^{+\infty} P(\phi_n^N \in B, M=k) + \sum_{k=n+1}^{+\infty} P(\phi_n^1 \in B, M=k) \tag{A.14}$$

Since $M(\omega)$ is proper,

$$0 \leq \sum_{k=n+1}^{+\infty} P(\phi_n^i \in B, M=k) \leq P(M>n) \xrightarrow[n \to +\infty]{} 0 \quad ; \quad i=1, N \tag{A.15}$$

In view of (A.15), taking appropriate limits in both sides of (A.14) proves the lemma.

Lemma A.4

Let $\{X_n \; ; \; n \epsilon Z\}$, $\{Y_n \; ; \; n \epsilon Z\}$ be stationary random processes independent of each other, and defined on the same probability space (Ω, F, P). Then,

$$P([(X_n, X_{n+1}, \ldots, X_{n+m_1-1}), (Y_k, Y_{k+1}, \ldots, Y_{k+m_2-1})] \in B) =$$

$$= P([(X_{n+n_1}, X_{n+n_1+1}, \ldots, X_{n+n_1+m_1-1}), (Y_{k+n_2}, Y_{k+n_2+1}, \ldots, Y_{k+n_2+m_2-1})] \in B)$$

for every $n, k, m_1, m_2, n_1, n_2 \in Z$, and every measurable set B in the induced probability space.

Proof: It is easy to see that the lemma holds if B is a finite disjoint union of measurable rectangles. An application of the monotone class theorem [16, Thm. 1.3.9] completes the proof.

_____

We are now ready to show the stability of the process ϕ_n defined in (A.5). To put the problem in the framework of regenerative processes, let z_n^1, z_n^2, $n \geq 1$, be the current lives of the session length processes $\{\bar{L}_k^1 ; k \geq 1\}$, $\{\bar{L}_k^2 ; k \geq 1\}$, respectively. Define

$$\bar{S}_n^1 = \begin{cases} \bar{S}_{\bar{N}_n^1}^1 & \text{if } n = \bar{R}_{\bar{N}_n^1}^1 \\ 0 & \text{Otherwise} \end{cases}$$

$$\bar{H}_n^2 = \begin{cases} \bar{H}_{\bar{N}_n^2}^2 & \text{if } n = R_{\bar{N}_n^2}^2 \\ 0 & \text{Otherwise} \end{cases}$$

Then,

$$\sum_{k=\bar{N}_m^1}^{\bar{N}_n^1} \bar{S}_k^1 = \sum_{i=m-\bar{z}_m^1}^{n} S_i^1$$

and

$$\sum_{k=\bar{N}_m^2+1}^{\bar{N}_n^2+1} \bar{H}_k^2 = \sum_{i=m}^{n-\bar{z}_n^2} H_i^2$$

The processes $\{(\bar{z}_n^1, \bar{S}_n^1) ; n \geq 1\}$, $\{(\bar{z}_n^2, \bar{H}_n^2) ; n \geq 1\}$ are regenerative with respect to the renewal processes $\{\bar{R}_k^1 ; k \geq 1\}$, $\{\bar{R}_k^2 ; k \geq 1\}$, respectively. The interrenewal times, i.e., the session lengths L_n^1, L_n^2, are aperiodic random variables. Furthermore, if

$\lambda_i < \lambda_i^*$, i=1,2, then $E(L_n^i) < \infty$, i=1,2. Therefore, there exist stationary versions

$\{((\bar{z}_n^1)^*, (\bar{S}_n^1)^*) ; n \geq 1\}$, $\{((\bar{z}_n^2, (\bar{H}_n^2)^*) ; n \geq 1\}$ of the processes $\{(\bar{z}_n^1, \bar{S}_n^1) ; n \geq 1\}$, $\{(\bar{z}_n^2, \bar{H}_n^2) ; n \geq 1\}$, respectively. Note that the above stationary versions are independent of each other, since the original processes are independent of each other.

From the regenerative theorem [7, Thm. 2], we have that

$$\lim_{n \to +\infty} \frac{1}{n} \sum_{i=1}^{n} \bar{S}_i^1 = \frac{E(\bar{S}_1^1)}{E(\bar{L}_1^1)} = p_1 \lambda_1 \qquad \text{a.e.} \qquad (A.16)$$

$$\lim_{n \to +\infty} \frac{1}{n} \sum_{i=1}^{n} \bar{H}_i^2 = \frac{E(\bar{H}_1^2)}{E(\bar{L}_1^2)} = d_2 \qquad \text{a.e.} \qquad (A.17)$$

where the last equalities in (A.16) and (A.17) follow from (14) and (16), respectively.

It follows from (A.16) that, for every fixed m,

$$\lim_{n \to +\infty} \frac{1}{n} \sum_{i=m-\bar{z}_m^{-1}}^{n} \bar{S}_i^1 = p_1 \lambda_1 \qquad \text{a.e.} \qquad (A.18)$$

Using standard renewal theory arguments it can be easily shown that

$$\lim_{n \to +\infty} \frac{1}{n} \bar{z}_n^2 = 0 \qquad \text{a.e.} \qquad (A.19)$$

From (A.17), (A.19) we have that

$$\lim_{n \to +\infty} \frac{1}{n} \sum_{i=m}^{n-\bar{z}_n^2} \bar{H}_i^2 = \lim_{n \to +\infty} \frac{n-\bar{z}_n^2}{n} \left(\frac{1}{n-\bar{z}_n^2} \sum_{i=m}^{n-\bar{z}_n^2} \bar{H}_n^2\right) = d_2 \qquad (A.20)$$

Define

$$Y_m^n = \sum_{i=m-\bar{z}_m^{-1}}^{n} \bar{S}_i^1 - \sum_{i=m}^{n-\bar{z}_n^2} \bar{H}_i^2 \qquad (A.21)$$

From (A.18), (A.20) and (17), we conclude that, for every m,

$$\lim_{n \to +\infty} \frac{1}{n} Y_m^n = p_1 \lambda_1 - d_2 < 0 \qquad (A.22)$$

From (A.22) and lemma A.3, we conclude, that, for every N and x,

$$\lim_{n \to +\infty} \inf P(\phi_n \le x) = \lim_{n \to +\infty} \inf P(\max(\max_{N \le m \le n} Y_m^n, 0) \le x) \tag{A.23}$$

where ϕ_n is as defined in (A.5) and Y_m^n is as defined in (A.21).

By theorem A.1 and lemma A.2, we have that for any $\varepsilon > 0$ there exists $N_0(\varepsilon)$, such that, for every $N \ge N_0(\varepsilon)$ and every n,

$$\left| P(\max(\max_{N \le m \le n} Y_m^n, 0) \le x) - P(\max((\max_{N \le m \le n} Y_m^n)^*, 0) \le x) \right| < \varepsilon \tag{A.24}$$

where $(\max_{N \le m \le n} Y_m^n)^*$ is the stationary version of $(\max_{N \le m \le n} Y_m^n)$; that is,

$$(\max_{N \le m \le n} Y_m^n)^* = \max_{1 \le m \le n-(N-1)} \left(\sum_{i=m-(\bar{z}_m^1)^*}^{n-(N-1)} (S_i^1)^* - \sum_{i=m}^{n-(N-1)-\bar{z}_{n-(N-1)}^2} (H_i^2)^* \right) \tag{A.25}$$

(In (A.24) we have used the fact that $(\bar{z}_m^1)^*$ is a proper random variable). Note that the ε in (A.24) does not depend on n, since the convergence in theorem A.1 is uniform. Thus, from (A.23), (A.24), and (A.25) we conclude that

$$\lim_{n \to +\infty} \inf P(\phi_n \le x) = \lim_{n \to +\infty} \inf P(\max(\max_{1 \le m \le n}(Y_m^n)^*, 0) \le x) \tag{A.26}$$

where

$$(Y_m^n)^* = \sum_{i=m-(\bar{z}_m^1)^*}^{n} (S_i^1)^* - \sum_{i=m}^{n-(\bar{z}_n^2)^*} (H_i^2)^*$$

Using lemma A.4 and the methodology of theorem 2 in [6,§3], it can be shown that

$$\lim_{n \to +\infty} P(\max(\max_{1 \le m \le n} (Y_m^n)^*, 0) \le x) = P(Y_\infty \le x) \tag{A.27}$$

where

$$Y_\infty = \max(\sup_{k \ge 0} \left(\sum_{i=-k-(\bar{z}_{-k}^1)^*}^{0} (S_i^1)^* - \sum_{i=-k}^{-(\bar{z}_0^2)^*} (H_i^2)^* \right), 0)$$

By the regenerative theorem we have that

$$\lim_{k \to +\infty} \frac{1}{k} \sum_{i=-k-(\bar{z}^1_{-k})^*}^{0} (S^1_i)^* = p_1 \lambda_1 \quad ; \quad \lim_{k \to +\infty} \frac{1}{k} \sum_{i=-k}^{-(\bar{z}^2_0)^*} (R^2_i)^* = d_2 \tag{A.28}$$

From (A.28) and the inequality $p_1 \lambda_1 - d_2 < 0$ we conclude that

$$\lim_{k \to +\infty} \left(\sum_{i=-k-(\bar{z}^1_{-k})^*}^{0} (S^1_i)^* - \sum_{i=-k}^{-(\bar{z}^2_0)^*} (R^2_i)^* \right) = -\infty \qquad \text{a.e.} \tag{A.29}$$

By (A.29) Y_∞ is proper. Therefore, from (A.26) and (A.27), we conclude that the process $\{\phi_n \; ; \; n \geq 1\}$ is stable. The proof of the theorem is now complete.

PROOF OF THEOREM 3

$Q^1_n, Q^2_n, B^1_n, B^2_n$ have limiting distributions as $n \to +\infty$, since they are measurable functions of Z_n, and $\{Z_n; n \geq 0\}$ is an irreducible and aperiodic Markov chain. Hence, by theorem 2, the processes $\{Q^1_n; n \geq 1\}$, $\{Q^2_n; n \geq 1\}$, $\{B^1_n; n \geq 1\}$, and $\{B^2_n; n \geq 1\}$ are stable.

Consider now the pointer position process $\{\bar{K}^1_n; n \geq 1\}$ in the dominant stack i. Since $\bar{K}^1_n$ is not affected by modification 2 in the algorithm, $\{\bar{K}^1_n; n \geq 1\}$ is regenerative with respect to the underlying renewal process $\{\bar{R}^1_k -; k \geq 1\}$. Thus, for $\lambda_i < \lambda^*_i$, $\{\bar{K}^1_n; n \geq 1\}$ is stable. By construction, $\bar{K}^1_n \geq K^1_n$ a.e. for every $n \geq 1$; therefore $\{K^1_n, n \geq 1\}$ is substable, and since K^1_n is a measurable function of Z_n, it is stable.

To show the ergodicity of $\{Z_n; n \geq 0\}$ we use the idea used in the proof of Theorem 1 in [8]. From the stability of the processes involved we have that for any $\epsilon > 0$ there exists an $x > 0$ such that

$$\lim_{n \to +\infty} \Pr(Q^i_n > x) < \epsilon/6 \quad , \quad i=1,2 \tag{A.30.a}$$

$$\lim_{n \to +\infty} \Pr(B^i_n > x) < \epsilon/6 \quad , \quad i=1,2 \tag{A.30.b}$$

$$\lim_{n \to +\infty} \Pr(K^i_n > x) < \epsilon/6 \quad , \quad i=1,2 \tag{A.30.c}$$

Now, using (A.30), we have

$$\lim_{n \to +\infty} \Pr(\{Q_n^1 \leq x\} \cap \{Q_n^2 \leq x\} \cap \{B_n^1 \leq x\} \cap \{B_n^2 \leq x\} \cap \{K_n^1 \leq x\} \cap \{K_n^2 \leq x\}) =$$

$$1 - \lim_{n \to +\infty} \Pr(\{Q_n^1 > x\} \cup \{Q_n^2 > x\} \cup \{B_n^1 > x\} \cup \{B_n^2 > x\} \cup \{K_n^1 > x\} \cup \{K_n^2 > x\}) \geq$$

$$1 - \lim_{n \to +\infty} \sum_{i=1}^{2} (\Pr(Q_n^i > x) + \Pr(B_n^i > x) + \Pr(K_n^i > x)) > 1 - \epsilon \qquad (A.31)$$

It is known that if $\{Z_n ; n \geq 0\}$ is not an ergodic chain, then for any finite subset V of the chain state space we have $\Pr(Z_n \in V) \to 0$ as $n \to +\infty$. Hence, by (A.31), $\{Z_n ; n \geq 0\}$ is an ergodic chain.

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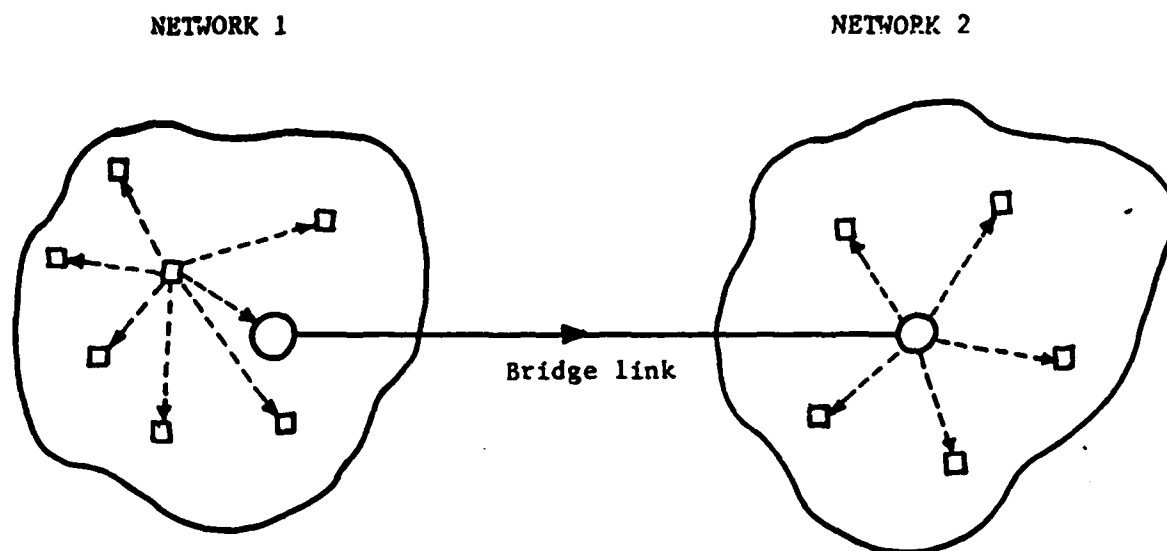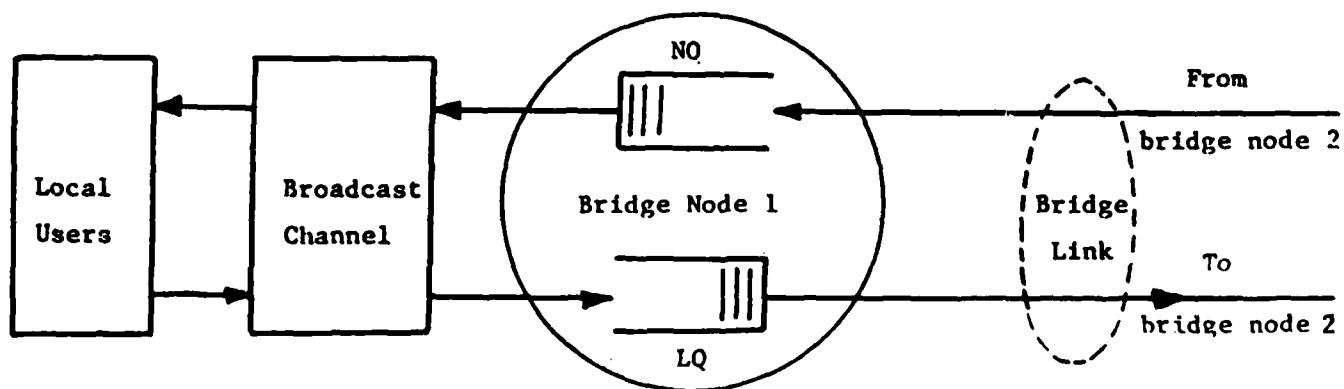[16] R.B. Ash, Real Analysis and Probability. New York: Academic Press 1972.

NETWORK 1   NETWORK 2

Bridge link

Figure 1

O Bridge node

☐ Network user

Figure 2

NO : Network Queue

LO : Link Queue

NETWORK 1

NETWORK 2

Q^1 : Queue 1
DQ^1: Distributed Queue i
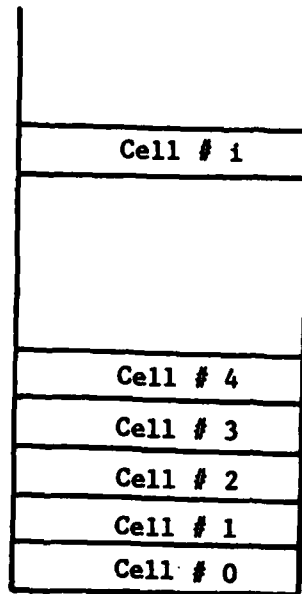BC : Broadcast Channel
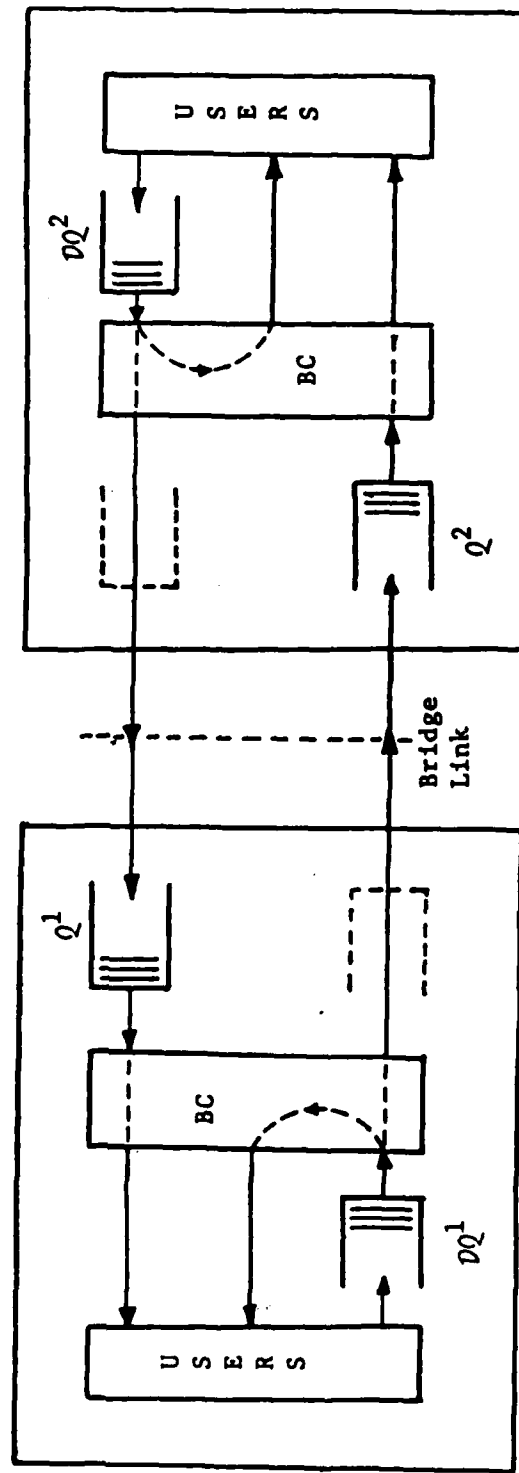
Figure 3

Flow of packets in the interconnected system
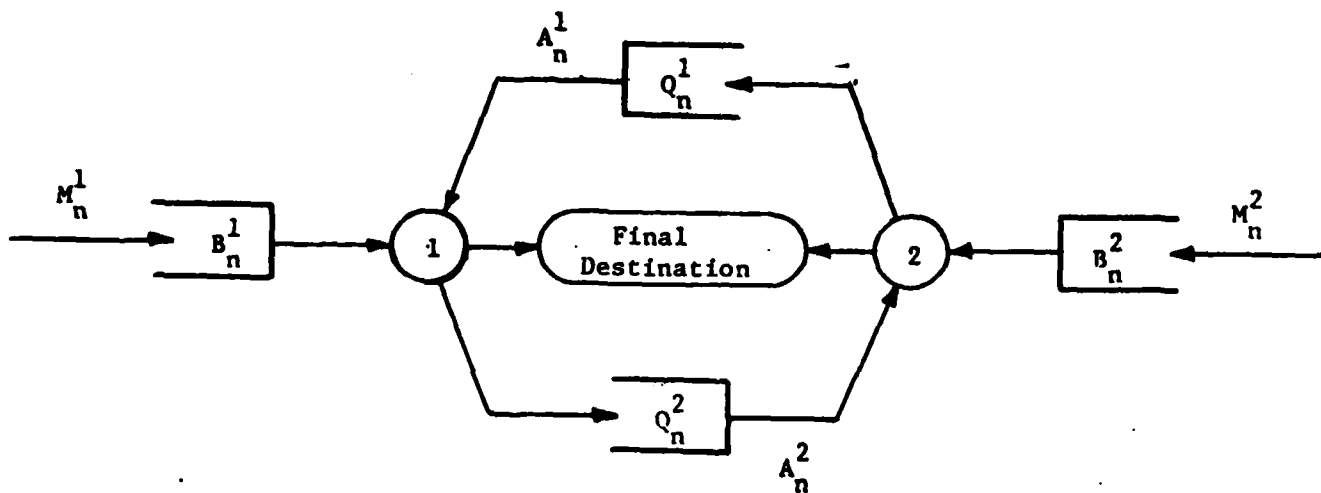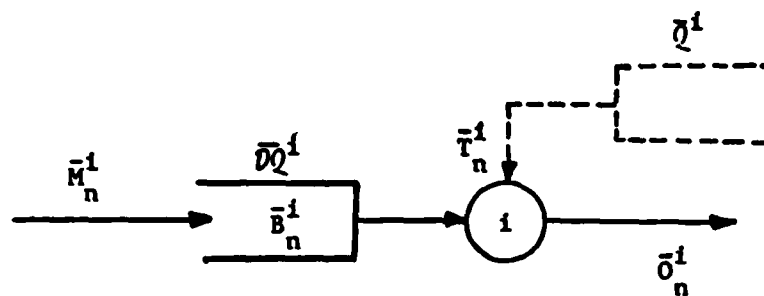
Figure 4

The Stack

Figure 5

Figure 6

Final
Destination
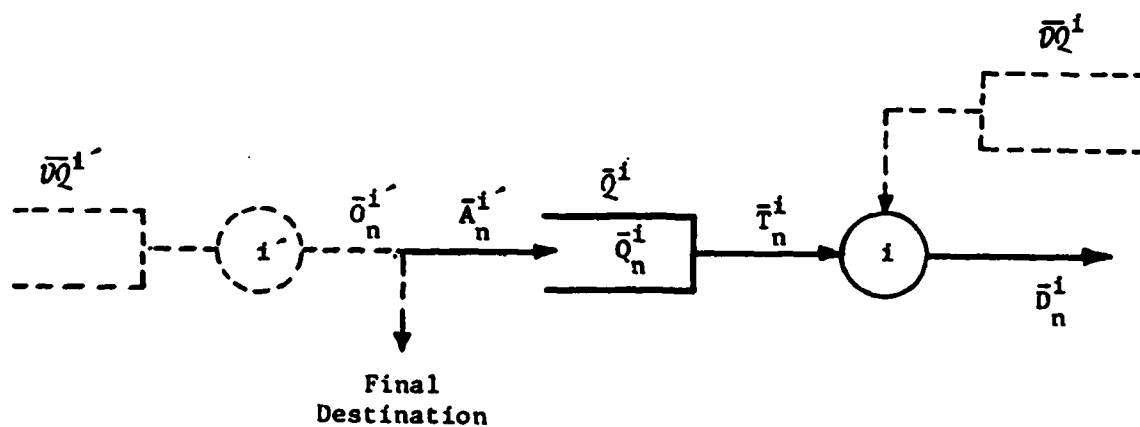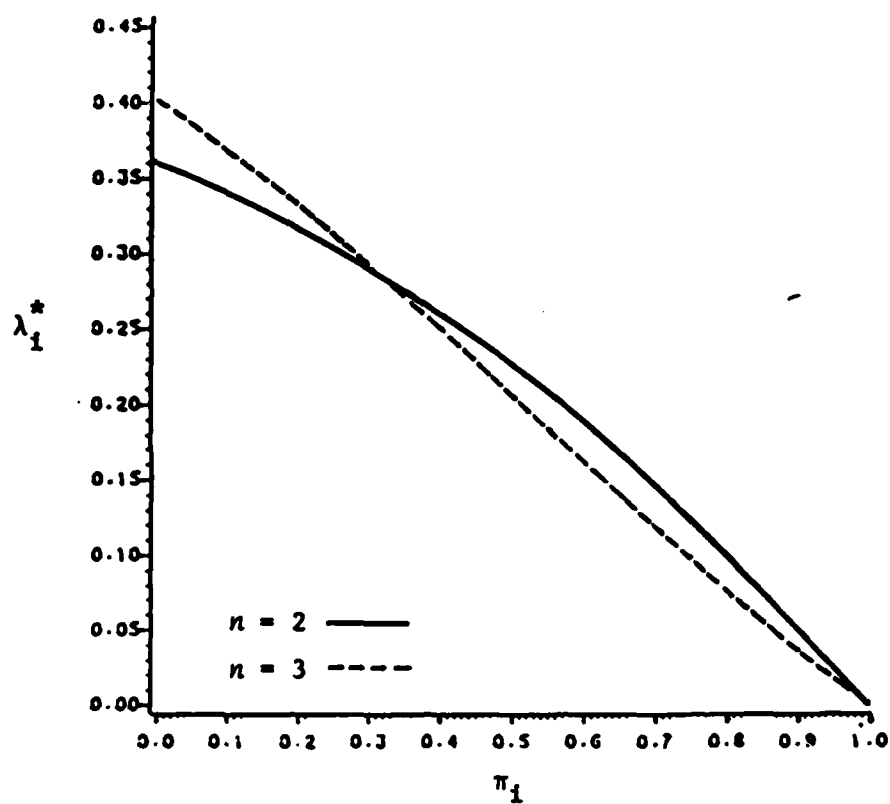
Figure 7

Figure 8

λ_1^* versus π_1 for $n = 2, 3$.
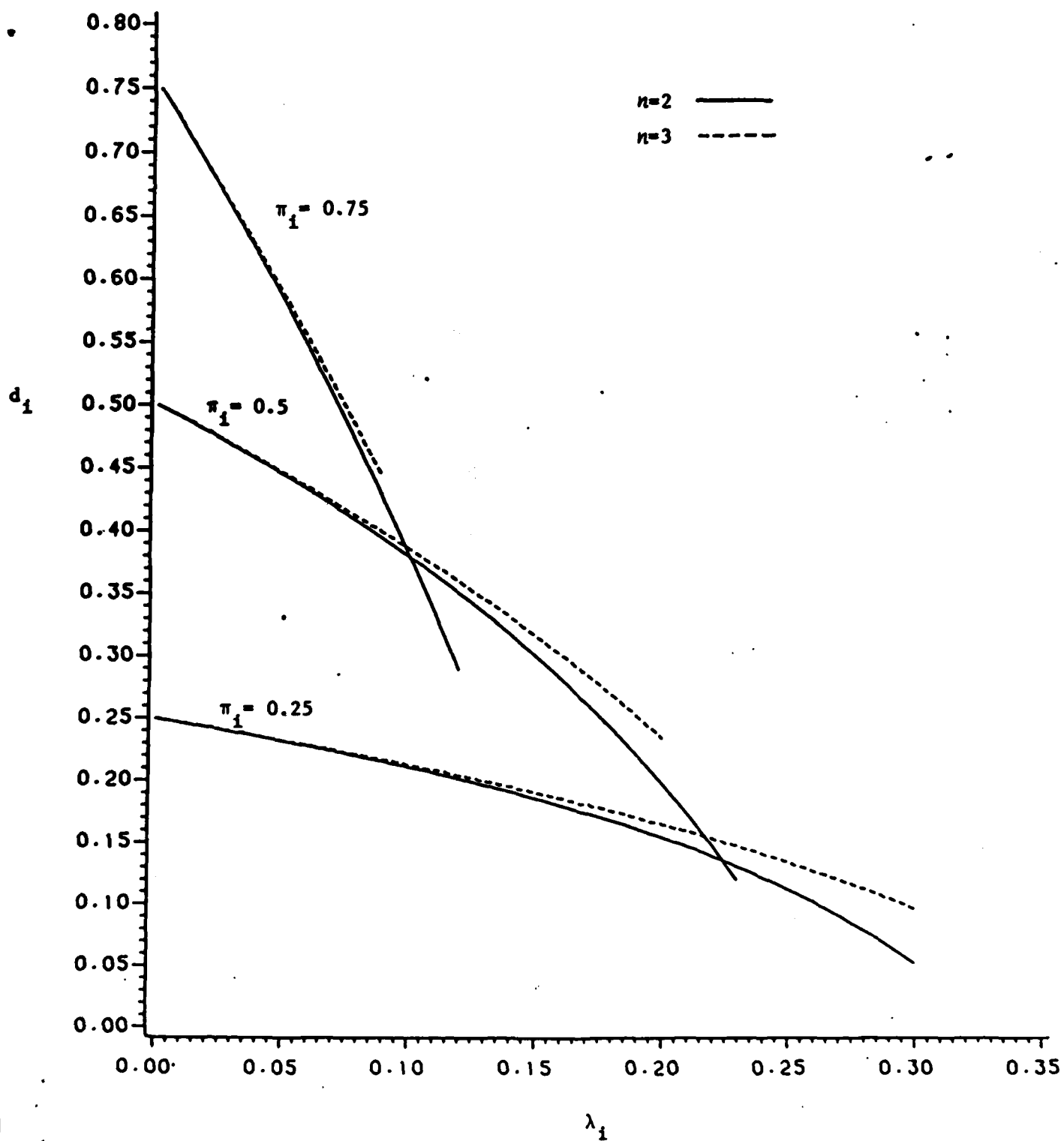
Figure 9

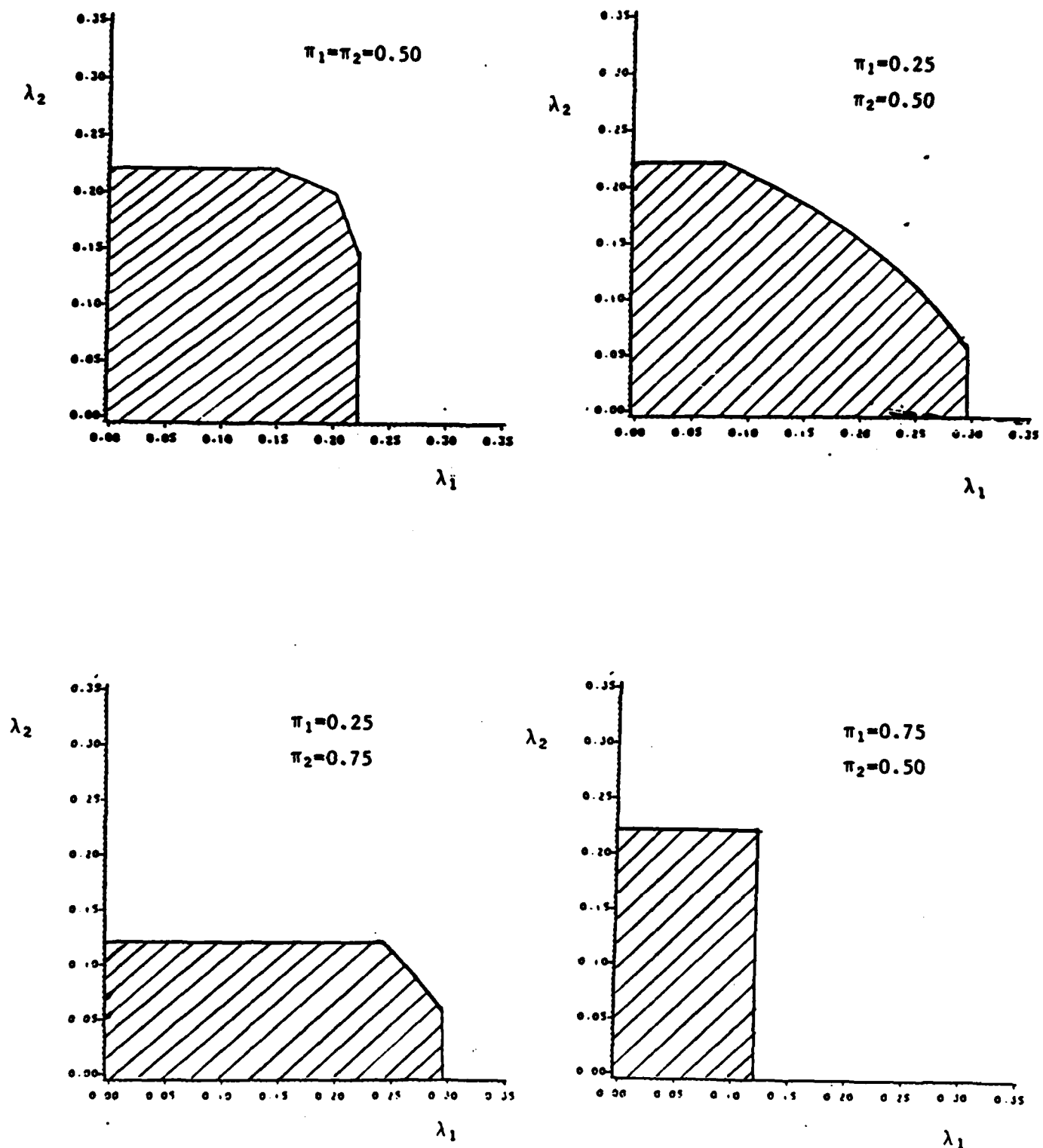d_i versus λ_i for $n=2,3$ and $\pi_i=0.25$, 0.50, 0.75.
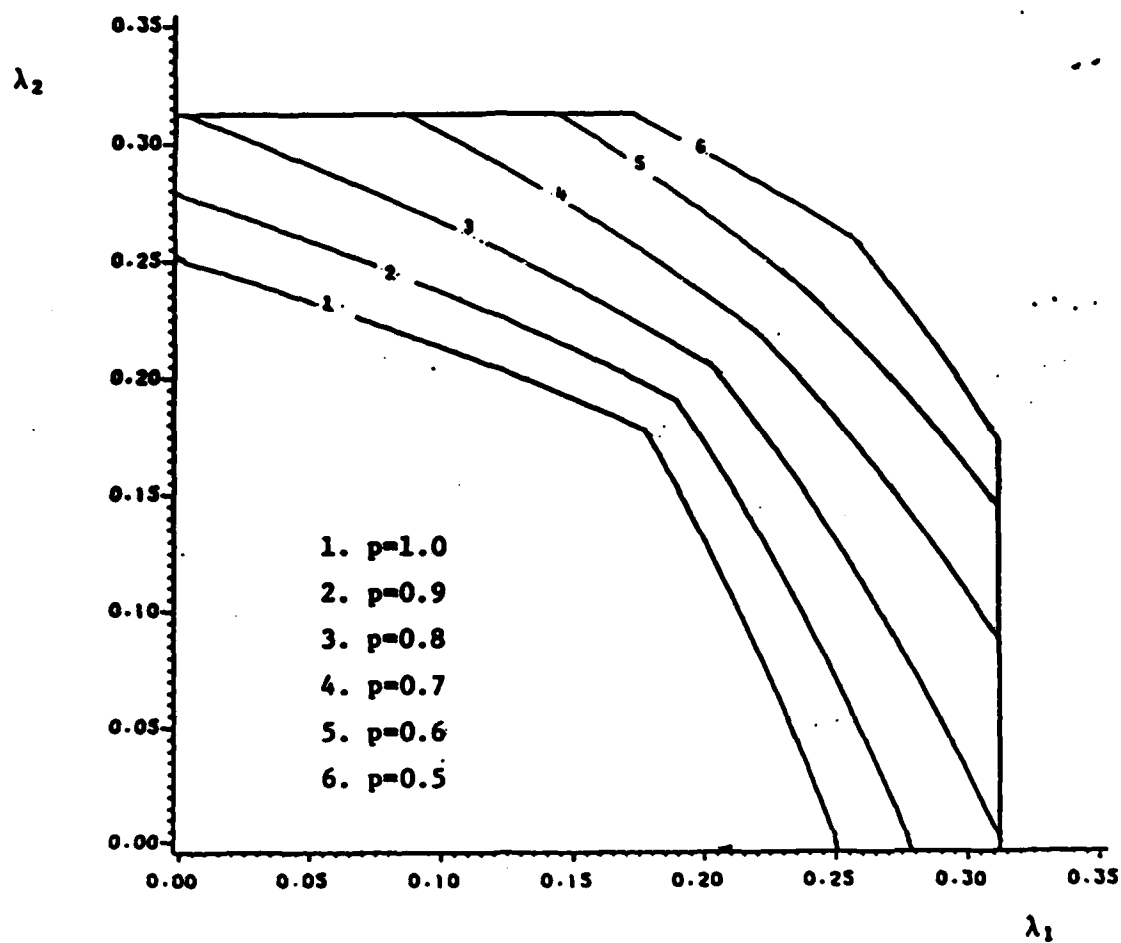
Figure 10

The stability subregion $\bar{S}$ under contention multiplexing, for $n=2$, $p_1=p_2=1$, and several values of π_1, π_2 .

Figure 11

The boundary of the stability subregion S under contention multiplexing, for $n=3$, $\pi_1=\pi_2=0.25$ and $p_1=p_2=p=0.5, 0.6, 0.7, 0.8, 0.9, 1$.

END

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DTIC